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Estimation of numerical accuracy for the flow field in a draft tube

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Abstract *The potential for overall efficiency improvements of modern hydro power turbines is a few percent. A significant part of the losses occurs in the draft tube. To improve the efficiency by analysing the flow in the draft tube, it is therefore necessary to do this accurately, i.e. one must know how large the iterative and the grid errors are. This was done by comparing three different methods to estimate errors. Four grids (122,976 to 4,592 cells) and two numerical schemes (hybrid differencing and CCCT) were used in the comparison. To assess the iterative error, the convergence history and the final value of the residuals were used. The grid error estimates were based on Richardson extrapolation and least square curve fitting. Using these methods we could, apart from estimate the error, also calculate the apparent order of the numerical schemes. The effects of using double or single precision and changing the under relaxation factors were also investigated. To check the grid error the pressure recovery factor was used. The iterative error based on the pressure recovery factor was very small for all grids (of the order* 10^{-4} *percent for the CCCT scheme and* 10^{-10}*percent for the hybrid scheme). The grid error was about 10 percent for the finest grid and the apparent order of the numerical schemes were 1.6 for CCCT (formally second order) and 1.4 for hybrid differencing (formally first order). The conclusion is that there are several methods available that can be used in practical simulations to estimate numerical errors and that in this particular case, the errors were too large. The methods for estimating the errors also allowed us to compute the necessary grid size for a target value of the grid error. For a target value of 1 percent, the necessary grid size for this case was computed to 2 million cells.*

Introduction

Proper runner design is the single most important step in the design of low head hydraulic turbines. The current design methods for the runner have reached a high level of refinement and make it possible to accurately predict the performance of the runner (Sottas and Ryhming, 1993). In the quest for better efficiency the attention is therefore today shifted towards the performance of other parts of the turbine system that are in contact with the water. One of the most important of these is the draft tube which is the curved diffuser that starts immediately after the runner and ends in the "tailrace" tunnel downstream of the power plant. In the draft tube a significant fraction of the total losses of the turbine system occurs (Raabe, 1984). The purpose of the draft tube (Figure 1) is to convert some of the kinetic energy of the flow from the runner into pressure

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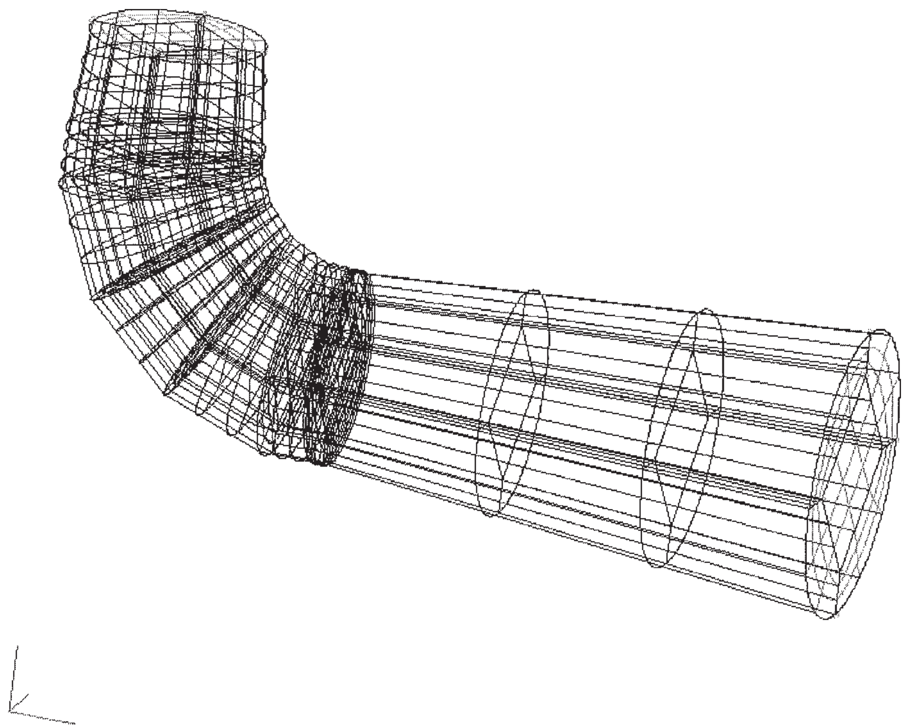

Figure 1.
Draft tube geometry

energy and thereby increase the efficiency of the turbine. It also guides the vertical flow immediately after the runner to a horizontal flow that can continue downstream. The flow into the draft tube has very little swirl, or streamwise vorticity, when the turbine is operating at best efficiency. However, it is not uncommon for the turbine to operate at other conditions than at its best efficiency point and in this case the flow will have a significant swirl.

The efficiency of a well designed turbine system is often as high as or higher than 93 percent (Dahlbäck, personal communication) which means that the potential for improvement of the overall efficiency is of the order of a few percent. This in its turn means that in order for a computer simulation to be useful it must be accurate to within a few percent, at least for quantitative predictions. For qualitative studies of the relative performance of different design options it may be acceptable with larger errors, as long as the trends are captured correctly. To achieve the necessary accuracy in a numerical prediction one must have an accurate mathematical model for the turbulent flow, i.e. the Reynolds stresses and be able to estimate the numerical errors in computations with this model.

The present paper presents an assessment of some methods to estimate errors in draft tube simulations objectively. The error estimates are also used to estimate the grid size that is necessary for accurate simulations of draft tube flows. The accuracy of turbulence models for the swirling flow in draft tubes will be the subject of a future paper.

The geometry of the draft tube was taken from the model turbine at the IMHEF laboratory at EPFL (Sottas and Ryhming, 1993). The flow in this draft tube has been analysed by several groups (Sottas and Ryhming, 1993) but none of them have done a formal error estimation.

Proposals for error estimation in complex flows are given by several authors: Celik and Zhang (1995); Ferziger (1993); Demuren and Wilson (1994); Wilcox (1993); Zingg (1992); Roache (1994); Celik *et al.* (1993); Ferziger and Peric (1996) and Celik and Karatekin (1997). In particular, the use of Richardson extrapolation (Celik and Zhang, 1995; Ferziger and Peric, 1996; Celik and Karatekin, 1997) opens up the possibility of both estimating errors and improving the results. However, there are situations in which Richardson extrapolation fails, e.g. when the reduction of the error due to grid refinement is non-monotonous or when the mesh is so coarse that the numerical errors do not decrease with decreasing mesh size in the way predicted by asymptotic analysis. Celik and Karatekin (1997), proposed a practical method for handling of cases with non-monotonous convergence. However, this method has only been applied to the case of a backward facing step and remains to prove that the method is valid in a general case. In the cases when Richardson extrapolation fails it is still useful as a warning that the solution needs further inspection before it can be trusted (Celik and Zhang, 1995).

The use of Richardson extrapolation has been thoroughly investigated for laminar flow (Ferziger, 1996) and has been found to give accurate results if the grid is sufficiently fine. It has also been applied to the two-dimensional flow over a backward facing step (Celik and Karatekin, 1997) where the method also gave an accurate extrapolation. However, this does not automatically imply that the method is useful for 3D turbulent flow with swirl in complex geometry. It is therefore the purpose of this paper to improve the confidence in the proposed methods for engineering type calculations by systematically investigating the flow in the draft tube geometry.

The use of wall function boundary conditions, which we have used in all computations presented below, gives rise to specific problems. One difficulty is that the grid refinement cannot be done all the way to the wall since the grid point closest to the wall has to be in the logarithmic region at $y^+ > 30$. However, if the near wall grid points are kept at a constant distance from the wall it is expected that Richardson extrapolation can still be used (Celik and Zhang, 1995). The advantage with wall functions is that the number of grid points can be reduced while the resolution of the internal flow is the same as with low Reynolds number versions of the turbulence models. An additional advantage with wall functions is that the convergence is better than with low Reynolds number modifications of the turbulence equations (Chen and Patel, 1988). The drawback is that the near wall behaviour of real flows sometimes does not conform to the law of the wall which is the basis for wall functions. However, whether the law of the wall is valid or not for draft tube flows is the subject of an ongoing study in our group and will be presented in a future paper.

The paper is organised with a summary of computational details in the next section followed by a discussion of iterative convergence and grid convergence. Finally, conclusions about the proper way to estimate errors are drawn based on the present results.

Computational details

A commercial code (AEA-CFX) was used for solving the draft tube flow (AEA Technology, 1995). It is a finite-volume based code using a structured non-staggered multi-block grid. The data transfer between blocks is done by the introduction of dummy cells outside the boundary of each block. This makes each block overlap a neighbouring block. The interior values in one block become the boundary conditions for the neighbour block and vice versa (AEA Technology, 1995).

All terms in all equations were discretised using second-order centred differencing (CDS) apart from the convective terms. The convective terms in the momentum equations were discretised using higher-order upwind differencing (HUW) (AEA Technology, 1995), which is a second-order method.

For the k and ε equations two different differencing schemes were tested, either hybrid differencing (HDS) or curvature compensated convective transport (CCCT). Hybrid differencing is formally only a first order scheme but it is widely used for engineering calculations due to its positive impact on convergence. It is therefore of interest to see what the penalty for the use of this method in the k and ε equations is. For the comparison we chose CCCT because it is second order and boundedness preserving (Gaskell and Lau, 1987).

For the pressure correction equation the formally second order accurate central differencing scheme (CDS) was used.

The expected overall behaviour of the numerical scheme is second order when CCCT is used in the k and ε equations. When hybrid differencing is used the expected overall behaviour is somewhere between first and second order.

The turbulence was modelled using the standard k-ε model and wall function boundary conditions although it is well known that this model is unable to represent all details of the flow accurately (Hanjalic, 1994). The argument for the use of this simple model in the present paper was that the main interest was to investigate methods for error estimation and that the k-ε model was believed to be sufficiently complex to give rise to similar numerical difficulties as a more complex model would do.

Six equations had to be solved: u, v and w-velocity, pressure correction, turbulent kinetic energy k and turbulent dissipation ε. The SIMPLEC algorithm was used for the pressure-velocity coupling (Van Doormal and Raithby, 1984). Different equation solvers, under-relaxation factors and differencing schemes were used depending on the equation (see Table I). The under-relaxation factors used are the default values for AEA-CFX (AEA

Technology, 1995) and these are believed to be sufficiently large to avoid unconverged solutions that appear to be converged due to heavy under-relaxation (more about this below).

Boundary conditions

The velocity and turbulent quantities were set at the inlet, the pressure and normal derivatives at the outlet and wall functions were used at the wall. The wall cell was for all mesh sizes chosen so that the outer control volume boundary was at $y^+ \approx 15$, to improve the accuracy of the wall function (Wilcox, 1993). The implementation of wall functions in AEA-CFX is done by solving the equation for the turbulent kinetic energy in the control volume immediately adjacent to the wall. The dissipation can then be calculated using wall functions. The velocity is finally obtained from the law of the wall by calculating the wall shear stress τ_w and the wall coordinate y^+. The tangential and normal velocities (Figure 2) were set according to measured values along one radius (Sottas and Ryhming, 1993) assuming axisymmetry at the inlet (Sottas and Ryhming, 1993). To apply these values to grid nodes at the inlet, linear interpolation was performed to the nodes. The turbulence quantities k and ε were set to constant values (AEA Technology, 1995) at the inlet calculated from the formulas

$$k_{inl} - c_{p1} u_{inl}^2 = 0.01230 \tag{1}$$

$$\varepsilon_{inl} = \frac{k^{3/2}}{c_{p2} D} = 0.01131 \tag{2}$$

where u_{inl} is the mean inlet velocity, c_{p1} and c_{p2} are empirical constants (AEA Technology, 1995) with values 0.002 and 0.3 respectively, and D is the outlet diameter of the runner. This means that a turbulent length scale of

$$C_\mu \frac{k^{3/2}}{\varepsilon_{inl}} = 0.025 \cdot D \tag{3}$$

Equation	Differencing scheme	Under relax. factor	Linear solver
u velocity	HUW	0.65	BLST
v velocity	HUW	0.65	BLST
w velocity	HUW	0.65	BLST
pressure	CDS	1.00	ICCG
k	HDS, CCCT	0.70	LRLX
ε	HDS, CCCT	0.70	LRLX

Note:
BLST = block stone, ICCG = preconditioned conjugate gradients, LRLX = line relaxation

Table I.
Differencing schemes, under-relaxation factors and solver methods for the linearised equations used in the computations

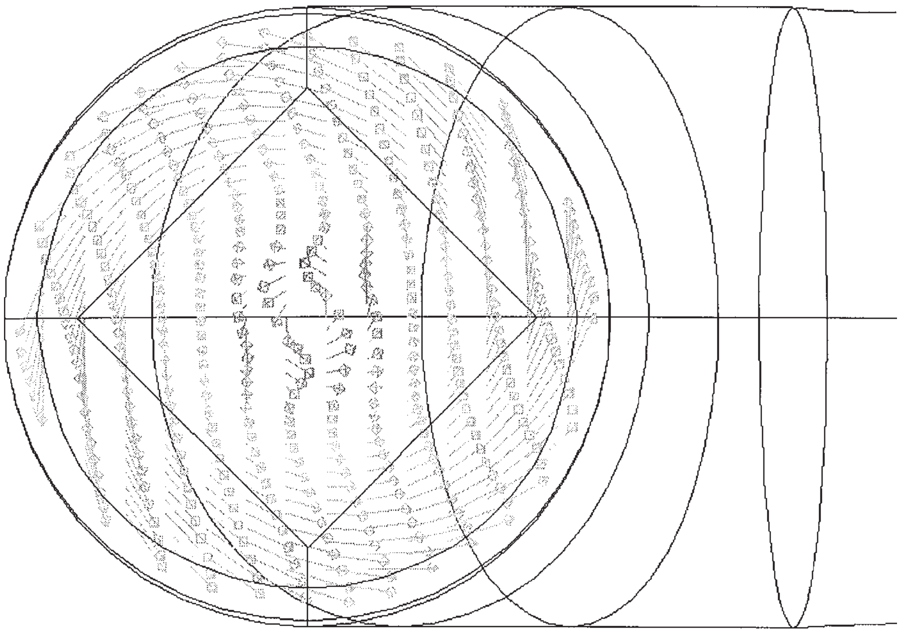

Figure 2.
Inlet swirling velocity distribution

or 2.5 percent of the runner outlet diameter and a turbulence intensity of

$$\sqrt{\frac{k_{inl}}{\frac{1}{2}u_{inl}^2}} = 6.3\% \tag{4}$$

are assumed.

At the outlet the pressure was set to a constant value and for all other variables a zero value of the first order derivatives at the outlet was adopted. The flow was therefore supposed to be fully developed at the outlet.

Computational grids

The draft tube calculations were performed on four different grids: a fine grid (grid 1), an intermediate grid (grid 3) having twice the grid cell size, a coarse grid (grid 4) having three times the grid cell size compared to the finest grid, and finally a grid between the fine and the intermediate (grid 2). The relevant numbers are listed in Table II.

Grid no.	No. of grid points	Cell size (equation (18))
1	122,976	1
2	79,079	1.16
3	15,372	2
4	4,592	2.99

Table II.
Grid and relative cell sizes

The grids were constructed with 40 blocks to properly resolve the sharp corners in the bend of the IMHEF (Sottas and Ryhming, 1993) model draft tube (see Figure 1). The circular cross sections were subdivided into five regions, with a central quadratic region, to avoid the degenerated control volumes at the outer circumference that appear when a mesh with only one region is used for a circular surface.

Iterative convergence

The iterative convergence error can be defined as the difference between the current and the exact solution to the discretised equations on the same grid (Demuren and Wilson, 1994). This error is difficult to define with a single global value. A common method to estimate the error is to utilise the residuals when the current solution is substituted into the discrete equations. One then often sums the absolute values of the residuals in all cells (the L_1 norm) (Ferziger and Peric, 1996) to get a global measure on the error. This value is called the absolute residual source sum or, colloquially, the residual.

A number of methods are available for estimating the iterative convergence error (Ferziger and Peric, 1996). They are all based on the assumption that a non-linear system of equations have an almost linear behaviour close to the converged solution. The task is then to estimate the spectral radius of the iteration matrix from the solution at different iteration levels. The underpinning mathematics of the method is flawless but the implementation in practice involves fine tuning of a few algorithmic parameters. We have therefore chosen to take a different route (described below) that involves no fine tuning but which involves more iterations.

The absolute residual source sum in the pressure correction equation can be physically interpreted as an artificial mass source. A small mass source corresponds to a solution that satisfies the continuity equation well. The mass source residual can be normalised with the total mass flow into the computational domain so that an objective measure of the relative error can be obtained.

For the other equations it is difficult to define an objective normalisation factor. Hence it is difficult to determine whether the solution has converged for all equations by reference only to the value of the absolute residual source sums. We have therefore adopted the procedure to inspect the whole convergence history in addition to the level of the residual source sum. Figure 3 shows a typical residual plot for a computation of the draft tube in Figure 1.

We have chosen to take the “knee” (indicated with an arrow in Figure 3) as a sign of convergence if at the same time the value of the residual source sum has dropped several orders of magnitude compared to its value after the second iteration. It is believed that the final value of the residual source sum depends on the discretisation scheme, the amount of under relaxation and on the machine precision. This assumption is made more credible by the observation that a change from double to single precision leads to an increased value of the mass residuals from $3.95*10^{-4}$ to $6.51*10^{-1}$ for grid 2 and the CCCT scheme (all

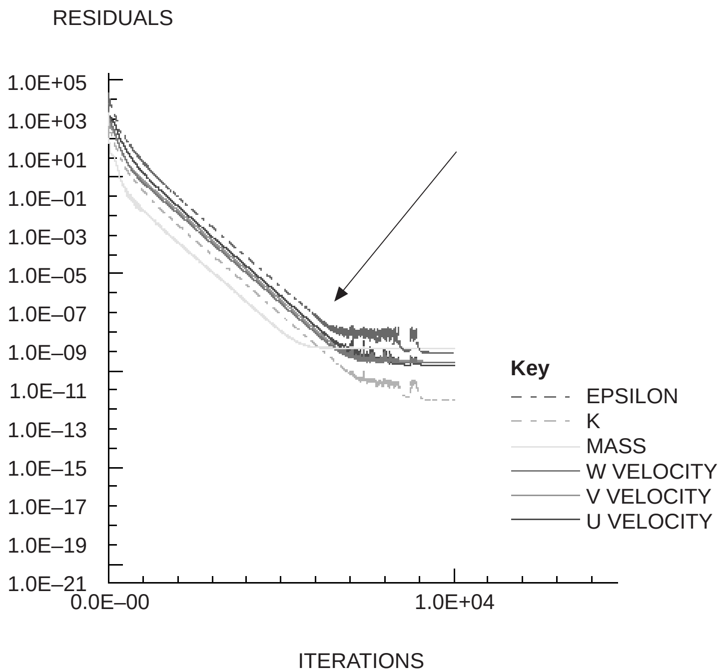

Figure 3. Residual plot for all equations for grid 1 (HDS, 122,976 cells). The arrow indicates the "knee" that is used as a sign of convergence. Notice that more than 5,000 iterations are necessary to reach convergence with this grid

other computations were done in double precision). A reduction of under relaxation factor in all equations with 10 percent for grid 2 and the CCCT scheme leads to a reduction of the mass residual from $3.95 \cdot 10^{-4}$ to $3.40 \cdot 10^{-5}$. It is also shown below how a change of differencing scheme affects the final level of the residuals (see Table III).

However, even if the "knee" criterion is satisfied convergence is not guaranteed (but very likely). To avoid "false convergence" we also monitored the values of all variables at a monitor point where the solution was believed to

Grid	Absolute mass source residual	Normalised mass source residual	Residual reduction factor
1 HDS	$1.28*10^{-9}$	$3.44*10^{-10}$%	$1.6*10^{12}$
2 HDS	$1.25*10^{-9}$	$3.36*10^{-10}$%	$1.4*10^{12}$
3 HDS	$1.24*10^{-9}$	$3.33*10^{-10}$%	$5.8*10^{11}$
4 HDS	$1.38*10^{-9}$	$3.71*10^{-10}$%	$5.8*10^{11}$
1 CCCT	$2.16*10^{-4}$	$5.81*10^{-5}$%	$9.2*10^{6}$
2 CCCT	$3.95*10^{-4}$	$1.06*10^{-4}$%	$4.5*10^{6}$
3 CCCT	$7.08*10^{-4}$	$1.90*10^{-4}$%	$1.0*10^{6}$
4 CCCT	$1.87*10^{-3}$	$5.03*10^{-4}$%	$4.3*10^{5}$

Note: The total mass flux into the draft tube was 372 kg/s. The residual reduction factor is the ratio of the residuals after the first and the last iteration

Table III. Mass source residuals for the four different grids using hybrid differencing or CCCT have strong gradients. When these monitor values stayed constant in at least six digits for several iterations at the same time as the previous convergence criterion was satisfied we considered the solution to be fully converged.

The resulting mass source residual when all rules above have been followed is summarised for the four computational grids in Table III. Notice that the normalised mass source residual is typically less than 10^{-9} for HDS. For the CCCT scheme the mass source residual after convergence is significantly higher. However, the continuity error is still negligible in comparison to the total flow into the computational domain.

The underlying reason for the differences in the final residuals between HDS and CCCT is outside the scope of the present paper. However, the numerical experiments described above indicate that the final level of the residuals are due to round off and cancellation errors. It is therefore likely that the difference is due to the detailed differences in the floating point operations that yields the converged (within the machine precision) solution.

Grid convergence

In this section three different error estimation methods are described. They will later be applied to the computational results.

Richardson extrapolation

Richardson extrapolation seems to be the most widely used method to estimate the grid convergence error. For example Ferziger (1993); Demuren and Wilson (1994); Wilcox (1993); Zingg (1992); Roache (1994) and Ferziger and Peric (1996) all use Richardson extrapolation to estimate the error in the solution. One interesting approach is proposed by Roache (1994) who introduces the Grid Convergence Index (GCI), which is based on Richardson extrapolation, to report grid refinement studies in CFD. The idea behind GCI is to relate the error for any grid refinement using any order of the method, to that for a grid doubling using a second order method, i.e. Roache (1994) suggests that grid doubling and second order methods should be the "standard" method to compare with. To use the GCI one has to perform two calculations, one on a fine grid and one on a coarse grid. The GCI is defined as (Roache, 1994):

$$GCI = \frac{3|\varepsilon|}{r^p - 1} \tag{5}$$

$$\varepsilon = \frac{f_2 - f_1}{f_1} \tag{6}$$

$$r = \frac{h_2}{h_1} \tag{7}$$

where r is the grid cell ratio between coarse (h_2) and fine grid (h_1), p is the order of the method used, h is the cell size, ε is the relative difference between the grids f_1 and f_2, the solutions from the fine and coarse grid, respectively. One interesting thing about the GCI, because it is based on Richardson extrapolation, is that it is applicable not only to grid values but also to solution functionals like efficiency and to plotted curves. A problem with GCI is that it becomes singular if the value f_1 is zero.

Another approach is presented by Celik and Zhang (1995). The exact relative grid convergence error is defined as:

$$e_r = \left| \frac{\phi_{\text{exact}} - \phi_h}{\phi_{\text{exact}}} \right| \tag{8}$$

where ϕ_{exact} is the exact value and ϕ_h is the value from a grid having grid cell size h. Because the exact value is not known one can use an extrapolated value as an approximation (Celik and Zhang, 1995), and define an approximate relative error.

$$e_{r,\text{approx}} = \left| \frac{\phi_{\text{extrapolated}} - \phi_h}{\phi_{\text{extrapolated}}} \right| \tag{9}$$

By using Richardson extrapolation or curve fitting it is possible to calculate $\phi_{\text{extrapolated}}$. The extrapolated value was first obtained as follows (Celik and Zhang, 1995). The error can, if the mesh is sufficiently fine, be expressed as:

$$\varepsilon_h = \phi_{\text{exact}} - \phi_h = a_1 h + a_2 h^2 + a_3 h^3 + \ldots \tag{10}$$

where h is the grid cell size and a_i are coefficients which can be functions of the coordinates (depending on the numerical scheme some of them might be zero) but do not depend on h in the asymptotic range. The error probably depends on h in a complex way if h is large due to the non-linearity of the governing equations. In that case many terms must be included in equation (10), but for sufficiently small h only the leading term matters:

$$\varepsilon_{\alpha h} = \phi_{\text{exact}} - \phi_{\alpha h} = C(\alpha h)^p \tag{11}$$

where α is the grid refinement factor (the grid cell ratio between the finest grid and the present grid), p is the order of the method and C is a coefficient that can be a function of the coordinates. By using equation (11) for three different grid refinement factors α_1 (=1 in most cases), α_2 and α_3, the following three equations for p, the extrapolated value and C can be derived:

$$\frac{\phi_{\alpha_2 h} - \phi_{\alpha_3 h}}{\phi_{\alpha_1 h} - \phi_{\alpha_2 h}} = \frac{\alpha_3^p - \alpha_2^p}{\alpha_2^p - \alpha_1^p} \tag{12}$$

$$\phi_{\text{extrapolated}} = \frac{\alpha_2^p \phi_h - \phi_{\alpha_2 h}}{\alpha_2^p - 1} \tag{13}$$

$$C = \frac{\phi_{\text{extrapolated}} - \phi_h}{h^p} \tag{14}$$

By first using equation (12) to check that the order p of the method is close to the expected value and then equation (13) (Richardson extrapolation) to get an approximation of the exact value, this will finally, by the use of equation (9), give an estimate of the grid convergence error.

Although equations (5) and (9) combined with equation (13) are derived in different ways, they are very similar. By rewriting equations (5) and (9) as:

$$\text{Equation (5)} \Longrightarrow GCI = \frac{3}{r^p - 1}\left|\frac{\phi_{rh} - \phi_h}{\phi_h}\right| \tag{15}$$

$$\text{Equations (9) and (13)} \Longrightarrow e_r = \left|\frac{\phi_h - \phi_{ah}}{\alpha^p \phi_h - \phi_{ah}}\right| \tag{16}$$

it is obvious that the two error estimates are normalised differences between the solutions on the two finest grids. The best estimate of the actual grid convergence error is probably obtained with equation (16). The GCI equation (15) is more conservative by having a "factor of safety". Notice the difference between r and α where r is the ratio between two adjacent grids in a sequence, whereas α is always referred to the finest grid in the sequence.

The cell size ratio indicator (α or r, cf. equations (15) and (16)) was calculated using

$$c = \left(\frac{N_1}{N_2}\right)^{1/3} \tag{17}$$

where N_1 is the number of control volumes for the fine grid and N_2 for the coarse grid (cf. Table II). This definition was chosen to make it possible to characterise the cell size with a single parameter so that equation (16) could be used. However, for this to be justified it is important that the distribution of grid points is geometrically similar (or almost) for all grids.

The error estimator used (equation (16)) also requires (because of the assumptions connected to Richardson extrapolation (Roache, 1994)) that the solutions are in the asymptotic range, i.e. the error must have the same variation between the grids as predicted by a Taylor-series analysis of the numerical scheme. Hence, a separate check should always be done that this is the case.

Curve fitting

The second method to obtain $\phi_{\text{extrapolated}}$ was least square curve fitting. The function used for the curve fitting was assumed to depend on the cell size in the same way as indicated by an asymptotic analysis of the discretisation scheme:

$$\phi = \phi_{\text{extrapolated}} + bh^p \tag{18}$$

where p is the order of the numerical scheme (for a second order scheme $p = 2$). The values from several computational grids are used together with a least squares fit to obtain values of the constants in equation (18). The resulting $\phi_{\text{extrapolated}}$ from the curve fitting was used in equation (9) to estimate the grid convergence error.

Grid convergence: results

The pressure recovery factor was used to check grid convergence. It is defined as (Agouzoul, 1990),

$$C_{pr} = \frac{P_{\text{out}} - P_{\text{in}}}{\frac{1}{2}\rho(u_{m,in}^2 + w_{m,in}^2)} \tag{19}$$

where P_{out} is the outlet static pressure (the outlet boundary condition was constant pressure), ρ is the density, $u_{m,in}$ is the mean inlet velocity and $w_{m,in}$ is the mean inlet swirl velocity. The mean inlet static pressure, P_{in}, is a direct result of the whole field solution for all variables. The pressure recovery factor indicates the degree of conversion of kinetic energy into static pressure (Agouzoul *et al.*, 1990) where a higher value means higher efficiency for the draft tube. The exact value of the pressure recovery factor depends on the whole field solution and can be seen as an integral property of the solution.

The pressure recovery factor differed greatly between grid 4 and the other grids, indicating that grid 4 is too coarse. Indeed, detailed inspection of the whole field solution (see Figure 4) revealed qualitative differences between the solution on grid 4 and the other solutions. Notice the absence of pressure extrema at the corners of the outer edge of the elbow in Figure 4 for the coarsest grid. The pressure recovery factor for the various grids and two different schemes (hybrid and CCCT) is shown in Table IV.

Notice that the difference between the results in Table IV with the two differencing schemes is in the third significant digit. However, a calculation of the apparent order of the numerical scheme (equation (12)) shows that the CCCT scheme has an order of about 1.6 while the hybrid scheme has an apparent order of 1.4 (see Table V). A calculation based on the coarsest grid yields a much lower order for both schemes, which indicates that the coarse grid results cannot be used for Richardson extrapolation.

The expected apparent order of the CCCT scheme is 2 (compared to the calculated value of 1.63 for the finest three grids), hence, it is likely that the grids are still slightly too coarse to yield an error that scales with the mesh size in the asymptotic way. An error estimate can still be computed but must be regarded as an approximation only. The computed error estimates for the pressure recovery factor (equation (19)) are shown in Table VI.

Based on estimate 1 in Table VI it appears that the relative error in the computed value of the pressure recovery factor with the finest grid is about 10 percent. This is a surprisingly large error since the finest grid had 122,976 cells. However, it should be kept in mind that the flow in the elbow draft tube takes

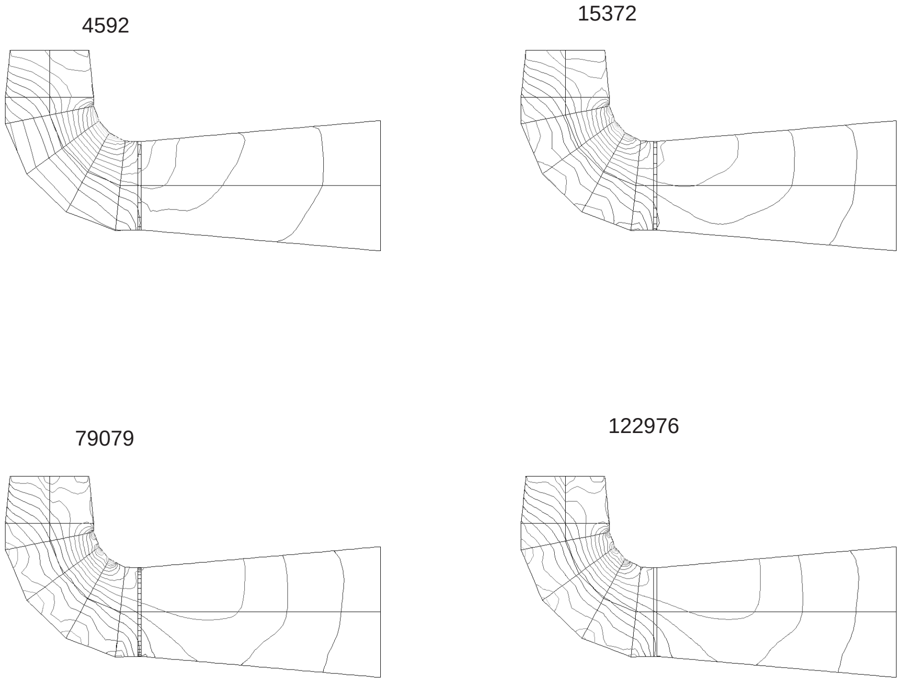

Figure 4.
Pressure contours in the symmetry plane for all grids. Notice the lack of local pressure extrema at the outer edge of the elbow for the coarsest grid

Table IV.
Pressure recovery factor for grid 1-4

Grid no.	C_{pr}/hybrid	C_{pr}/CCCT
1	0.4051	0.4044
2	0.4155	0.4146
3	0.4808	0.4832
4	0.5609	0.5621

Table V.
Apparent order from equation (12) of the numerical scheme for different grid combinations

Grid combination	Order p/hybrid	Order p/CCCT
1,2,3	1.44	1.63
1,3,4	1.12	1.02
2,3,4	1.08	0.95

Table VI.
Estimated relative error in the pressure recovery factor with three different error estimation methods

Method	Estimate 1, equations (9) (12) (13)	Estimate 2, GCI equation (5)	Estimate 3 equations (9) (18), grid 1,2,3, using p = 2
Fine grid/hybrid	0.12	0.33	0.063
Fine grid/CCCT	0.10	0.28	0.067

place in a very complicated geometry (sharp corners in the bend) and exhibits a very complex behaviour (swirl, streamline curvature and separation). The complete resolution of all small scale features of the mean flow will require a very large number of grid points.

Having seen that 122,976 cells are too few to resolve all features of the flow one may ask how fine the mesh should be to yield an acceptably small error. If the error is set to a target value, of 1 percent, equation (19) can be used to estimate the necessary grid refinement to about 2 million cells. This grid size was too fine to allow us to compute the flow with our available computer resources. However, ongoing work on a more powerful parallel computer will allow us to check this in the near future.

Conclusions

This paper presents an assessment of methods to estimate numerical errors in complex three-dimensional flows. The methods can be divided into two parts, estimates of iterative convergence error and estimates of grid convergence errors.

Iterative convergence error was estimated by inspection of plots of the absolute residual source sums for all equations (see Figure 3). As a sign of convergence it was required that the residual curves should exhibit an initial significant decrease by several orders of magnitude followed by a relatively constant level of the residual sums. Numerical experiments with variations of solution parameters and differencing scheme indicate that the resulting solution is converged within machine precision.

The proposed iterative convergence error estimate is more conservative than the methods based on estimates of the spectral radius of the iteration matrix (Ferziger and Peric, 1996). The present method is also much easier to implement but the price for the convenience is that extra work has to be spent to get to a converged solution.

The grid convergence error was estimated using Richardson extrapolation and curve fitting. To use Richardson extrapolation as an error estimator at least three grids are necessary so that a reliable test of whether the solutions are in the asymptotic range can be made. Applied to the pressure recovery factor this resulted in a grid error of about 10 percent for the finest grid. The apparent order of the scheme was in this case 1.6 instead of the expected value of 2.0. This indicates that the sequence of grids was too coarse to be considered as being in the asymptotic region where the grid error is proportional to the mesh size to some power only. An estimate of the number of cells that would give an error of less than 1 percent in the pressure recovery factor shows that the grid should have at least 2 million cells.

Although the computations appear to have been done with a mesh that was too coarse to be ideally suited for the Richardson extrapolation based error estimators it appears that the methods are useful in practical situations. If the apparent order of the scheme is not close to the theoretical value one should regard the error estimate with caution. However, the results can still be used to compute the necessary grid refinement that would give an acceptable error.

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